Title

A geologically constrained basis for global inversion of CSEM data using a dynamic number of parameters

Authors

Ole-Johan Skrede[1], Jan Petter Morten[2], Linus Boman[3]

Abstract

In this paper, we introduce a new parameterization of the model space, where the basis is constrained on a priori information about the geology. The parameterization is able to represent complex model structures using only a few parameters, which can significantly reduce the computational complexity of the inversion problem. This facilitates a global inversion approach, and we consider a simulated annealing optimization. In order to facilitate the search for a minimum-parameter representation, we extended the inversion to be able to optimize for a dynamically varying number of variables. We demonstrate the method by inversion of marine CSEM data from the Troll West Oil Province. The algorithm is able to recover a resistivity profile which agrees with well log data from the area. The dimensionality of the parameter space is reduced by more than an order of magnitude using our approach compared to a layer-based discretization of the resistivity. The physically feasible models obtained are attributed to the constrained basis which makes the inversion very robust.

[1]EMGS
[2]EMGS
[3]EMGS

Introduction

The scale of the numerical problems involved in geophysical inversion of state-of-the-art datasets, *e.g.* 3D marine CSEM (Controlled-Source ElectroMagnetic) data, can be so large as to prohibit the use of global optimizations. Less computationally demanding local optimization approaches are therefore used. At the same time, the geological complexity in the areas where the technology is deployed can make the processing and imaging challenging. Often, the local optimization inversion approaches need good initial models to succeed. The construction of these models can require a lot of manual effort.

The global optimization approach is desirable since it will allow to start inversion with minimum knowledge of the geology and find a best fitting model. The computational cost increases with the number of iterations, which in turn increases with number of inversion parameters Therefore, a model representation with fewer parameters is desirable. Typically, global optimizations are only possible assuming 1D geology to reduce the number of parameters for CSEM data inversion. Due to the complexity of real geologies and the resolution of 3D CSEM data we expect, however that local optimization schemes in 3D that can tackle a large numbers of parameters will be required to achieve an acceptable final imaging result from the survey. The main use of global inversions assuming lower dimensionality is then to extract larger resistors and background trends to populate initial models for 3D inversion.

We introduce a new parameterization of the model parameter space for the inversion problem. It is applied on inversion of CSEM data, but the general idea also applies in other problem instances. The main idea in our approach is that the parameters of the model should correspond to the parameters of functions that each correspond to expected geological structure. The resulting reduction in degrees of freedom can dramatically reduce the computational cost of inversion algorithms. However it is important that the parameterization is compatible with the main features of the data which we aim to explain. In order to accommodate the potential complexity of the geology, we study a trans-dimensional algorithm (Ray and Key, 2012). If the initial parameterization is too restricted to describe the geology, the number of parameters may increase to allow a more complex model representation.

Theory

The objective of CSEM data inversion is to find a sub-surface resistivity model that explains the observed data. We denote the vector of the observed data $\mathbf{d}^{\text{obs}}$, the model $\mathbf{m}$, and the predicted data $\mathbf{d} = \mathscr{F}(\mathbf{m})$. Here we introduced the forward operator $\mathscr{F}$ determined by Maxwell's equations. The measured data $\mathbf{d}^{\text{obs}}$ is the electric and magnetic fields recorded by seafloor receiver nodes. The best model is found by an optimization routine which for some norm $||\cdot||$ minimizes $||\mathbf{d} - \mathbf{d}^{\text{obs}}||$.

A plane layer (1D) model is considered in the forward modelling, with sub-surface layers and interfaces stacked as $[\rho_0, z_1, \rho_1, \ldots, \rho_{N_l-1}, z_{N_l}, \rho_{N_l}]$. The water resistivity ρ_0 and water depth $z_{wd}(= z_1)$ are assumed known and constitute fixed parameters. The model discretization has variable layer thickness. The initial spacing is set to $z_2 - z_1 = 5\,\text{m}$, and then increases so that the number of interfaces is $N_l = 150$ and the depth of the last interface is at $z_{N_l} = 2500\,\text{m}$ in the considered case. This variation is set according to the expected resolution of the data. For such 1D models, it is possible to use a fast quasi-analytic evaluation of the resulting electromagnetic field components due to an electromagnetic dipole source (Løseth and Ursin, 2007). The approach studied in this paper does not consider the layer resistivities of the discretization as the unknown parameters of the inversion. Rather, we propose a parameterization of the resistivity values in terms of a limited set of basis functions, where each basis function corresponds to a property of the geology.

We denote the 1D resistivity model as $\rho(z;\mathbf{x})$, where $\mathbf{x}$ are the basis function parameters (the free parameters to be optimized) and the total number of free parameters in the parameterization is denoted N. The form of the functions is based on prior knowledge about the general geology beneath the seabed, with the purpose of keeping the number of parameters low whilst maintaining a detailed representation of the resistivity profile. The parameterization should fulfill two criteria, (1) each parameter should have direct relevance to a geological property or structure, and (2) the building blocks of the parameterization should correspond to specific physical responses. We expect that the level of parameter reduction using this scheme, compared to a straightforward discretization with the layer resistivities as free parameters, will be significant. Also, since the basis is restricted using a priori information about the subsurface geology, it gives an intuitive and geophysically meaningful representation.

The proposed parameterization of the subsurface consists of three types of terms: a polynomial, sets of Heaviside step functions, and Gauss functions, described by $(z > z_{wd})$

$$\rho(z;\mathbf{x}) = \underbrace{\sum_{n_p=0}^{N_p} c_{n_p}(z-z_{wd})^{n_p}}_{\rho_p(z;\mathbf{x}_p)} + \underbrace{\sum_{n_r=1}^{N_r} \rho^r_{n_r}[H(z-z^r_{n_r}) - H(z-z^r_{n_r}-t)]}_{\rho_r(z;\mathbf{x}_r)} + \underbrace{\sum_{n_g=1}^{N_g} \frac{\beta_{n_g}}{\sqrt{2\pi w^2_{n_g}}} \exp\left\{\frac{-(z-z^g_{n_g})^2}{2w^2_{n_g}}\right\}}_{\rho_g(z;\mathbf{x}_g)}. \tag{1}$$

The polynomial terms $\rho_p(z;\mathbf{x}_p)$, are determined by the coefficients c_{n_p} which constitute free parameters $\mathbf{x}_p = (c_0,\ldots,c_{N_p})^T$, resulting in a total of N_p+1 parameters to be determined for this part. The polynomial will represent background model trends, and is included mostly because of the first two terms which describe a resistivity shift of the whole model, c_0 and a *compression trend* c_1 (Zhang, 2011).

The second group of terms, $\rho_r(z;\mathbf{x}_r)$ with $\mathbf{x}_r = (\rho^r_1, z^r_1, \ldots, \rho^r_{N_r}, z^r_{N_r})^T$, are composed from step functions H, and are used to model thin resistors in the model. These have resistivity $\rho^r_{n_r}$ and thickness t. Due to the transverse resistance equivalence, we fix the thickness in order not to introduce non-unique parameters. The value $z^r_{n_r}$ determines the upper boundary (*i.e.* the depth) of resistor number n_r. This results in $2N_r$ parameters to be optimized. In terms of the physical response, these terms will generate the guided wave which is utilized in CSEM hydrocarbon exploration.

The final group of terms, $\rho_g(z;\mathbf{x}_g)$ with $\mathbf{x}_g = (\beta_1, w_1, z^g_1, \ldots, \beta_{N_g}, w_{N_g}, z^g_{N_g})^T$, is composed of Gaussians where β_{n_g} is an amplitude amplifying factor, w_{n_g} is a measure of the width of the bell curve, and $z^g_{n_g}$ is the depth at which the bell curve is centered. This sums up to $3N_g$ free parameters. The Gaussian terms are included to represent lithological variations that are captured as smooth variations. We include these terms as smooth functions though the geology they represent typically consists of layers with sharp boundaries. The reason is that the low-frequency CSEM data will only be able to resolve a smooth reconstruction of such a layer. In this way, the parameterization is compatible with the data resolution, and we emphasize that the data responses captured by these terms are of different nature than $\rho_r(z;\mathbf{x}_r)$.

The parameters that determine the resistivity model in Equation (1) is $\mathbf{x} = (\mathbf{x}_p^T, \mathbf{x}_r^T, \mathbf{x}_g^T)^T$. The number of terms in each group of terms is limited to avoid overfitting, and the total number of free parameters is $N = 1 + N_p + 2N_r + 3N_g$. That is, the optimization is done for the N inversion parameters, instead of one parameter for each of the $N_l = 150$ layers. The parameters are bounded in order to maintain their direct relevance to a geological property. The bounds are defined by minimum and maximum values, and for some parameters (*e.g.* resistivity) the minimum value is required to be positive.

The problem of finding the best configuration of parameter values is stated as an optimization problem,

$$\rho(z)^\star = \arg\min_{\mathbf{x}\in D} \varepsilon(\rho(z;\mathbf{x})), \tag{2}$$

where the space of parameter values $D \subset \mathbb{R}^N$ is bounded by the parameter constraints, and ε is the cost function (misfit functional) to be minimized. The misfit functional ε is the complex squared, weighted L^2-norm of the difference between the the observed and synthetic inline electric field data E_x,

$$\varepsilon(\rho) = \sum_{\mathbf{r}_s,f} \frac{\left|E_x^{Obs}(\mathbf{r}_s,\mathbf{r}_r,f) - E_x^{Synth}(\mathbf{r}_s,\mathbf{r}_r,f|\rho)\right|^2}{\alpha^2|E_x^{Obs}(\mathbf{r}_s,\mathbf{r}_r,f)|^2 + \eta^2}, \tag{3}$$

where the sum runs over all source positions $\mathbf{r}_s$ and transmitter frequencies f. The constant factor α represents a relative uncertainty caused by navigation fluctuations and instrument calibration inaccuracy, and η represents an ambient noise contribution (Morten and Mittet, 2012). Regularization is usually required to achieve realistic inversion results, however in this case the restriction in model space resulting from the explicit parameterization guarantees that the inverted model is geologically feasible. We can therefore carry out inversion without regularization and still achieve geologically meaningful results.

In order to solve the inverse problem Equation (2) we use simulated annealing (Kirkpatrick et al., 1983). Although originally applied on combinatorial problems with a fixed dimensionality, the algorithm can be extended to cope with a varying number of continuous optimization parameters. A new proposed dimensionality is sampled at each iteration, together with a new proposed solution which is uniformly sampled from a feasible neighbourhood of the current solution. For the results presented here, only one parameter is updated at each iteration, and the cooling schedule is a monotonic decreasing, reciprocal function with respect to the iteration number (Lundy and Mees, 1986). By adjusting the number of parameters, the algorithm can adjust the model complexity to the minimum required to describe the data and achieve lower absolute misfit compared to using a fixed number of parameters. The optimizations were terminated after 20 000 iterations, but typically converged earlier.

Results

The method outlined above has shown promising results for both synthetic and real data. We will here present representative results using data from the Troll West Oil Province (Gabrielsen et al., 2009). In Figure 1 observed and final model synthetic data are shown, both amplitude and phase, using three transmitter frequencies (0.25,0.75, and 1.25 Hz). Only the recordings from one receiver are inverted, with a source towed towards the receiver with offset from 1000 m to 10 000 m with a source spacing of 100 m. The water resistivity is 0.271 Ω m and the water depth at the receiver location is 325.6 m. As seen in the figure, the inverted data match the observed data well, at least up to 7 km offsets. The inversion

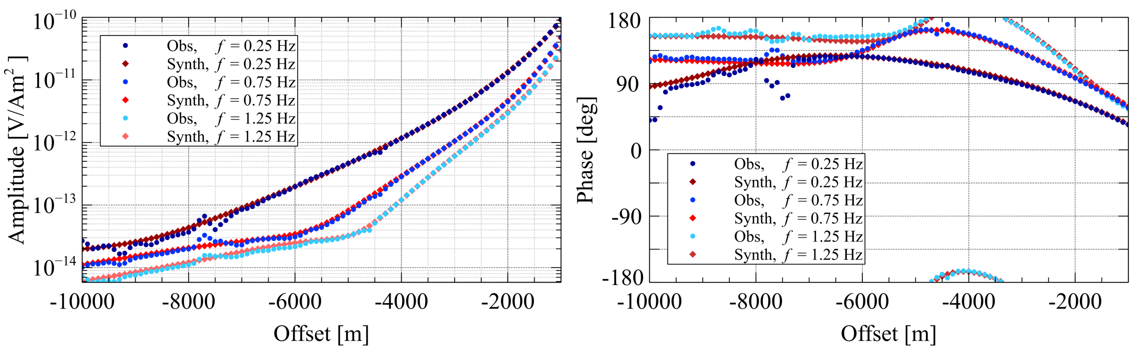
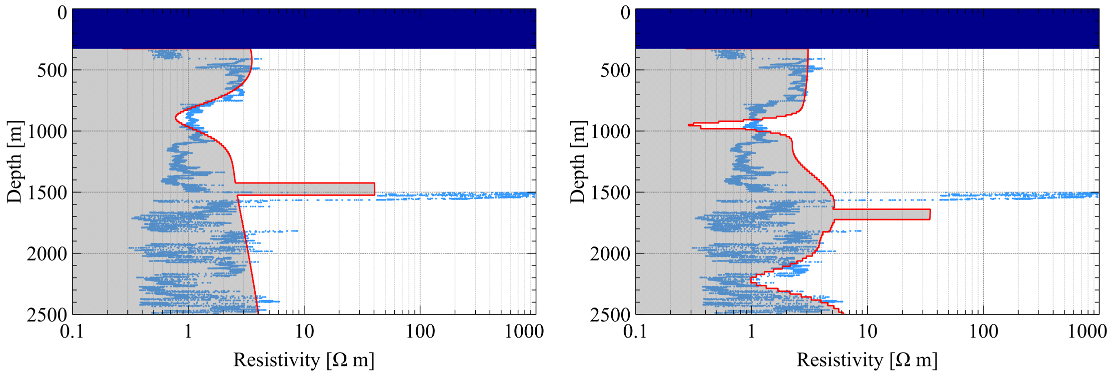


Figure 1 *Inline E_x field data from Troll West Oil Province, dynamic number of parameters.*

results are shown in Figure 2 as resistivity profiles, where Figure 2(a) is obtained with a fixed number of inversion parameters ($N = 7$), and Figure 2(b) shows results from a trans-dimensional inversion where the number of inversion parameters is $N_p = 1$, $N_r = 8$, $N_g = 5$. It should be noted that some of the coefficents are very small and the corresponding term in Equation (1) could be removed without any significant effect on the synthetic data. Blue dots are horizontal resistivity data from a nearby well log, which is used as reference for the resistivity profile from inversion (red line). The inversion results show good agreement with the well log, and the main features of the well log is captured by the inversion. We note that the CSEM inversion recovers the vertical resistivity rather than the horizontal resistivity measured in the well log, and we expect the two measurements to differ in an anisotropic shale. It is straightforward to extend the present inverse scheme to recover the horizontal and vertical resistivity of an anisotropic model, but the inline data from a single receiver typically does not have good sensitivity towards horizontal resistivity. The thin resistive layer in the inversion result is located close to the



(a) Fixed dimensions, final RMS = 2.47

(b) Dynamic dimensions, final RMS = 2.26

Figure 2 *Two representative inversion results, in (a) with 7 (fixed) parameters, and in (b)* $N_p = 1$, $N_r = 8$, $N_g = 5$ *(dynamic) parameters. The root mean square (RMS) of the misfit is shown in the captions.*

depth of the corresponding hydrocarbon reservoir resistor in the well log data. Since the thickness of the Heaviside function is fixed and is thicker than the resistor in the well log data, due to transverse resistance equivalence it is less resistive. Both cases identify the resistor and the dip in resistivity at about 800 m depth, and the transdimensional inversion also locates the reduced resistivity at 2200 m. Compared to a parameterization in the 150 layer resistivities, the considered approach achieves a 20× to 10× reduction in number of parameters.

Conclusion

We introduce a new parameterization of the model space with the potential to greatly decrease the number of free parameters. This can improve the performance of stochastic inversion schemes that seek the global minimum, or even make such schemes feasible in higher dimensions. The basis for the parameterization are the parameters of functions that conform to geological variation and geophysical response characteristics from a priori information. The trans-dimensional inversion scheme can adjust the number of parameters dynamically to ensure that a minimum number of parameters is used to describe the data. We demonstrate the approach by inversion of CSEM field data in a 1D global inversion scheme, and achieve a realistic resistivity profile in good agreement with well log data. We achieve more than an order of magnitude reduction of model dimensionality compared to a layer-based parameterization.

Acknowledgements

We thank EMGS ASA for permission to publish this work.

References

Gabrielsen, P.T., Brevik, I., Mittet, R. and Løseth, L.O. [2009] Investing the exploration potential for 3d csem using a calibration survey over the troll field. *First Break*, **27**, 67.

Kirkpatrick, S., Gelatt, C. and Vecci, M. [1983] Optimization by simulated annealing. *Science, New Series*, **220**(4590), 671.

Løseth, L. and Ursin, B. [2007] Electromagnetic fields in planarly layered anisotropic media. *Geophysical Journal International*, **170**, 44.

Lundy, M. and Mees, A. [1986] Convergence of an annealing algorithm. *Mathematical programming*, **34**, 111.

Morten, J.P. and Mittet, R. [2012] Detection and imaging sensitivity of the marine CSEM method. *Geophysics*, **77**(6), E411.

Ray, A. and Key, K. [2012] Bayesian inversion of marine CSEM data with a trans-dimensional self parameterizing algorithm. *Geophysical Journal International*, **191**, 1135.

Zhang, J. [2011] Pore pressure prediction from well logs: Methods, modifications, and new approaches. *Earth-Science Reviews*, **108**, 50.