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Low-frequency electromagnetic fields in applied geophysics: Waves or diffusion?

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ABSTRACT

Low-frequency electromagnetic (EM) signal propagation in geophysical applications is sometimes referred to as diffusion and sometimes as waves. In the following we discuss the mathematical and physical approaches behind the use of the different terms. The basic theory of EM wave propagation is reviewed. From a frequency-domain description we show that all of the well-known mathematical tools of wave theory, including an asymptotic ray-series description, can be applied for both nondispersive waves in nonconductive materials and low-frequency waves in conductive materials. We consider the EM field from an electric dipole source and show that a common frequency-domain description yields both the undistorted pulses in nonconductive materials and the strongly distorted pulses in conductive materials. We also show that the diffusion-equation approximation of low-frequency EM fields in conductive materials gives the correct mathematical description, and this equation has wave solutions. Having considered both a wave-picture approach and a diffusion approach to the problem, we discuss the possible confusion that the use of these terms might lead to.

INTRODUCTION

Electromagnetic (EM) methods have been used for a long time and for different purposes in applied geophysics (see, for example, the extensive treatment in Nabighian, 1987). The EM methods in geophysics are based on the theory of classic electrodynamics in conductive materials, which is treated in well-known works on EM theory (Stratton, 1941; Adler et al., 1960; Jackson, 1998; Griffiths, 1999; Kong, 2000; Ulaby, 2001) and optics (see the chapter on metal optics in Born and Wolf, 1999). Ward and Hohmann (1987) give a comprehensive review of the theory for geophysical applications.

Since its introduction as a hydrocarbon-exploration technique about five years ago, seabed logging (SBL) has become an important complementary tool to seismic exploration methods in the detection and characterization of possible hydrocarbon-filled layers in sedimentary environments. SBL is a variety of controlled-source electromagnetic (CSEM) sounding that uses an electric dipole source and array of seabed receiver antennas in a manner suggested by Cox et al. (1971) and Young and Cox (1981). SBL exploits a kind of guiding of EM energy that occurs in resistive layers located in a more conductive environment (Eidesmo et al., 2002; Ellingsrud et al., 2002).

Eidesmo et al. (2002) refer to EM signal propagation as both diffusion and waves. Furthermore, they talk about flowing inductive and galvanic currents as well as an equivalent picture of a respective TE and TM mode of EM-field propagation (Born and Wolf, 1999). In a variety of geophysical literature it is common to refer to the propagation of EM fields in conductive media as diffusion. Spies (1989) discusses the depth of penetration of various EM sounding experiments with different source signatures. The propagation is referred to as diffusion; both transient signals and a related time-domain diffusion depth in addition to single-frequency components and their skin depths are considered. Raiche and Gallagher (1985) use the concept of a diffusion velocity for transient EM signals in the conductive earth, and Lee et al. (1989) consider a fictitious wavefield representation of the diffusive EM field. Virieux et al. (1994) refer to EM signal propagation in the earth as a diffusion process, whereas Nekut (1994) discusses ray-trace tomography for low-frequency fields in a conductive earth, a well-known wave-theory technique. He refers to the propagation of the fields as diffusive EM waves. Ward and Hohmann (1987) elaborate on wave propagation of fields in conductive media. They further refer to the differential equations

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that describe EM fields in conductive media as both diffusion equations and wave equations.

The propagation of low-frequency EM fields in conductive media is sometimes referred to as diffusion and sometimes referred to as waves. It might be interesting to ask if one naming convention is better than the other. What is the motivation for referring to the propagation as waves, and what is the motivation for the diffusion picture? Is there any physical understanding connected to the words that calls for some care when using either of the terms? In the following we show that propagation of EM fields in conductive materials is well described within the framework of the standard theory of EM wave propagation. We demonstrate that the wave equation in the frequency domain contains the diffusion-like equation in the high-loss approximation, and that this equation has wave solutions attenuated as shown by Ward and Hohmann (1987). Moreover, we find no sharp boundary between propagation of fields in a wave-like manner and a diffusion-like manner. Based on these observations, both terms can be used. However, it is interesting to discuss both the diffusion and the wave picture connected to low-frequency EM fields in conductive materials. We consider the simple cases of an electric dipole in a homogeneous nonconductive medium and a conductive medium and in both cases consider harmonic and transient source signatures. During the discussion, it will become evident that picturing low-frequency propagation of EM fields in conductive media as both waves and diffusion calls for some caution.

We first review Maxwell's equations in the frequency domain. The purpose of the basic mathematical review is to show that a common frequency-domain description contains both the physics behind high-frequency propagation in nonconductive regions and low-frequency propagation in conductive regions. We introduce a complex wavenumber and impedance before we derive inhomogeneous wave equations that describe EM wave propagation in inhomogeneous media. The reason for treating the equations in the frequency domain becomes evident as we compare the frequency-domain and time-domain differential equations. The vector wave equation without damping term is a hyperbolic partial differential equation, and the vector diffusion equation is a parabolic partial differential equation (Sommerfeld, 1967; Davison and Doeschl, 2004). A vector wave equation with a damping term is one of these types, depending on the material parameters involved. In the frequency domain, all partial differential equations reduce to an elliptic equation. We review the simple plane-wave solutions of the frequency-domain wave equation and discuss the relation between the electric and magnetic field and the wavenumber's dependence on frequency for these waves. The theory for reflection and refraction of plane waves at an interface between two homogeneous media is reviewed in Appendix A.

Next, we look at approximate solutions of the inhomogeneous wave equations by applying asymptotic ray theory, which is well known and extensively used in fields such as optics (Born and Wolf, 1999), seismics (Červený and Hron, 1980), and ocean-wave modeling (Svendsen and Jonsson, 1976). The eikonal equation that we derive from our ray-solution ansatz is complex and frequency dependent. We show that it becomes real and frequency independent for the two extreme cases of nondispersive waves in nonconductive materials and highly dispersive, low-frequency waves in conductive materials. Even if ray theory is a high-frequency approach, it is also applicable in conductive media for low frequencies, as shown by Nekut (1994).

We briefly consider the standard theory of wave propagation in horizontally layered media (Brekhovskikh, 1960; Wait, 1962; Chave and Cox, 1982; Ursin, 1983; Born and Wolf, 1999) and simplify to homogeneous media where the wave equations are solved in terms of the Green's functions. We calculate the Green's functions and use them to derive an expression for the radiated EM field from an electric dipole source. We consider dipole radiation in homogeneous media and show that the simple frequency-domain description gives a nondispersive wave in nonconductive materials and an attenuated wave in conductive materials for a time harmonic source current. When regarding a radiated pulse, we get the simple pulse propagation in nonconductive materials and the highly dispersive, distorted, diffusion-like pulses in conductive materials. Thus, as expected, we get a separate behavior depending on the dispersion relation for the two extreme cases. Having presented a unified mathematical treatment of classic electrodynamics, we discuss some basic differences between a diffusion picture and a wave picture of the propagation of fields in conductive regions.

ELECTROMAGNETIC FIELDS IN THE FREQUENCY DOMAIN

We introduce the Fourier transform pair

$$\boldsymbol{\Psi}(\omega) = \int_{-\infty}^{\infty} \boldsymbol{\psi}(t)\exp(i\omega t)\mathrm{d}t,$$

$$\boldsymbol{\psi}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \boldsymbol{\Psi}(\omega)\exp(-i\omega t)\mathrm{d}\omega, \tag{1}$$

where t denotes time, ω denotes angular frequency, $\boldsymbol{\Psi}$ is a field vector in the frequency domain, and $\boldsymbol{\psi}$ is a field vector in the time domain. Let $\mathbf{E}(\mathbf{r},\omega)$ represent the complex electric field and $\mathbf{H}(\mathbf{r},\omega)$ the complex magnetic field. The EM fields in the time and frequency domains are interrelated by the Fourier transform pair defined in equation 1. We restrict our discussion to linear and isotropic media throughout this paper. The constitutive relation between the electric displacement $\mathbf{D}$ and the electric field $\mathbf{E}$ then becomes $\mathbf{D} = \tilde{\varepsilon}(\mathbf{r},\omega)\mathbf{E}$, where $\tilde{\varepsilon}$ is the scalar complex electric permittivity, which includes a possible conductive property of the medium. The relation between the magnetic induction $\mathbf{B}$ and magnetic field $\mathbf{H}$ becomes $\mathbf{B} = \mu(\mathbf{r},\omega)\mathbf{H}$, where μ is the scalar magnetic permeability. In the constitutive relations we neglect the possible nonlocal effects in space of the material parameters since these effects normally become important above optical frequencies. The complex EM field obeys Maxwell's equations, which become (see Stratton, 1941; Jackson, 1998)

$$\nabla \cdot (\tilde{\varepsilon}\mathbf{E}) = \rho_0, \tag{2a}$$

$$\nabla \cdot (\mu\mathbf{H}) = 0, \tag{2b}$$

$$\nabla \times \mathbf{E} = i\omega\mu\mathbf{H}, \tag{2c}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_0 - i\omega\tilde{\varepsilon}\mathbf{E}, \tag{2d}$$

where $\mathbf{J}_0$ is a source current density and ρ_0 is a source charge density. The macroscopic averages of the EM properties of the surrounding medium are described by μ and $\tilde{\varepsilon}$ only. The local conduction current density normally is described well by Ohm's law, $\mathbf{J}_c = \sigma\mathbf{E}$, where σ is electric conductivity. The displacement current is given as $\mathbf{J}_d = -i\omega\varepsilon\mathbf{E}$, where ε is electric permittivity. The term $\mathbf{J} = -i\omega\tilde{\varepsilon}\mathbf{E}$ includes both the conduction current and the displacement current, and the complex electric permittivity becomes $\tilde{\varepsilon} = \varepsilon + (i\sigma/\omega)$. This way of denoting the conductivity is consistent with the electron model of Drude (Jackson, 1998). It is a matter of convention whether one writes the ohmic term as a standalone term or in combination with the dielectricity. The charge-conservation equation becomes $i\omega\rho_0 = \nabla \cdot \mathbf{J}_0$, where $\mathbf{J}_0 \neq 0$ only at the source antenna.

The material properties can be expressed by two secondary parameters that characterize the interaction of the EM field at a specific frequency with the properties of the medium. These are the complex wavenumber,

$$k = \omega\sqrt{\mu\tilde{\varepsilon}} = \sqrt{\omega^2\mu\varepsilon + i\omega\mu\sigma}, \tag{3}$$

and the characteristic impedance,

$$\eta = \sqrt{\frac{\mu}{\tilde{\varepsilon}}} = \sqrt{\frac{\mu}{\varepsilon + \frac{i\sigma}{\omega}}}. \tag{4}$$

From Maxwell's equations the wave equations for inhomogeneous media can be derived:

$$\nabla^2\mathbf{E} + k^2\mathbf{E} + \nabla\left[\mathbf{E}\cdot\nabla\left(\ln\tilde{\varepsilon}\right)\right] + i\omega\nabla\mu\times\mathbf{H} = -i\omega\mu\left[\mathbf{J}_0 + \frac{\nabla(\nabla\cdot\mathbf{J}_0)}{k^2}\right] + \frac{\nabla\cdot\mathbf{J}_0}{i\omega}\nabla\left(\frac{1}{\tilde{\varepsilon}}\right), \tag{5a}$$

$$\nabla^2\mathbf{H} + k^2\mathbf{H} + \nabla\left[\mathbf{H}\cdot\nabla\left(\ln\mu\right)\right] - i\omega\nabla\tilde{\varepsilon}\times\mathbf{E} = -\nabla\times\mathbf{J}_0. \tag{5b}$$

Plane waves

To understand EM wave propagation, it is useful to consider plane waves. Many problems involving spherical waves or cylindrical waves can be simplified by expanding the waves into a spectrum of plane waves (Sommerfeld, 1909; Weyl, 1919), and in asymptotic theory the assumption of local plane waves can provide useful simplifications. Expressions for the reflection and transmission of EM fields at boundaries are easily derived if we restrict ourselves to plane waves at planar boundaries (see Appendix A). Here, we look at some elementary properties of EM plane waves and consider a source-free homogeneous medium ($\mathbf{J}_0 = 0$). Wave equations 5a and 5b simplify to

$$\nabla^2\boldsymbol{\Psi} + k^2\boldsymbol{\Psi} = 0, \tag{6}$$

where $\boldsymbol{\Psi}$ can represent either the electric or the magnetic field. The wavenumber k is given by the dispersion relation in equation 3, where the medium parameters in this case are space invariant. Equation 6 has the plane-wave solution

$$\boldsymbol{\Psi}(\mathbf{r}) = \boldsymbol{\Psi}_0\exp(ik\hat{\mathbf{s}}\cdot\mathbf{r}), \tag{7}$$

where $\hat{\mathbf{s}}$ is the unit vector in the direction of propagation. Maxwell's equations 2c and 2d imply that, in source-free regions, the vectors $\hat{\mathbf{s}}$, $\mathbf{E}$, and $\mathbf{H}$ form a right-handed, orthogonal system and are interrelated by

$$\mathbf{E}(\mathbf{r}) = -\eta\hat{\mathbf{s}}\times\mathbf{H}(\mathbf{r}), \quad \mathbf{H}(\mathbf{r}) = \frac{1}{\eta}\hat{\mathbf{s}}\times\mathbf{E}(\mathbf{r}). \tag{8}$$

Thus, the electric and magnetic fields are transverse to each other and the direction of propagation. Moreover, the wavenumber k is complex if the medium is conductive, and in this case the EM wave is attenuated in the direction of propagation in homogeneous regions.

We find it illustrative to consider the wavenumber's dependence on frequency, permittivity, and conductivity. Assuming for simplicity that the material parameters are independent of frequency, we express the wavenumber in terms of a phase velocity c_p and attenuation coefficient β. Then $k = (\omega/c_p) + i\beta$, where β often is given in terms of a skin depth $\delta = 1/\beta$. From the dispersion relation in equation 3, we see that the phase velocity and attenuation coefficient have the same frequency dependence:

$$c_p = \frac{1}{\sqrt{\mu\varepsilon}}f\left(\frac{\omega}{\omega_0}\right), \quad \beta = \frac{1}{2}\sigma\sqrt{\frac{\mu}{\varepsilon}}f\left(\frac{\omega}{\omega_0}\right), \tag{9}$$

where $\omega_0 = \sigma/\varepsilon$ is the characteristic frequency at which the magnitude of the displacement current equals that of the conduction current and

$$f(x) = x\sqrt{2\left(\sqrt{1+\frac{1}{x^2}} - 1\right)} \cong \begin{cases} 1 & \text{for } x \gg 1, \\ \sqrt{2x} & \text{for } x \ll 1. \end{cases} \tag{10}$$

This function is illustrated in Figure 1. The asymptotic limits are seen to be very good for $x > 10$ and $x < 0.1$, respectively. The plane wave can be expressed in terms of phase velocity and attenuation factor as $\boldsymbol{\Psi}(r) = \boldsymbol{\Psi}_0\exp(-\beta r)\exp(i\omega r/c_p)$, where $r = \hat{\mathbf{s}}\cdot\mathbf{r}$. In the time domain a time harmonic plane wave at frequency ω now becomes

$$\boldsymbol{\psi}(r,t) = \boldsymbol{\psi}_0 e^{-\beta r}\cos\left[\omega\left(\frac{r}{c_p} - t\right)\right]. \tag{11}$$

We observe that the attenuation increases and the phase velocity decreases with increasing conductivity. Moreover, the attenuation and phase velocity are seen to be frequency dependent except when $\omega\varepsilon/\sigma \gg 1$.

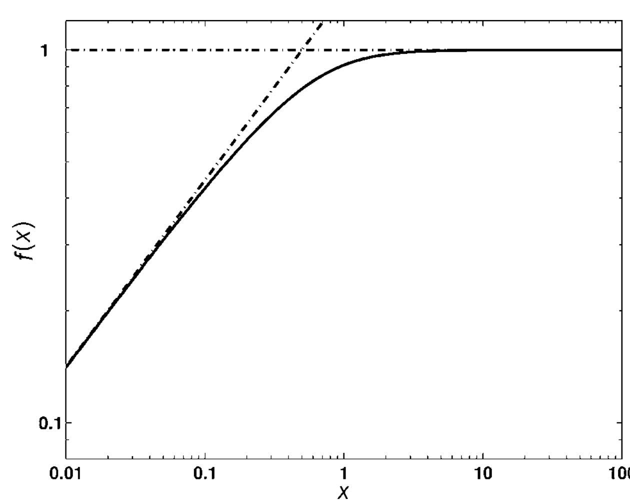

Figure 1. The function $f(x)$ defined in equation 10 in log-log scale. The asymptotic limits for $x \gg 1$ and $x \ll 1$ are very good for $x > 10$ and $x < 0.1$, respectively.

Asymptotic ray theory

In inhomogeneous media, asymptotic ray-series solution methods are an alternative to pure numeric methods for modeling the EM fields described by Maxwell's equations. In ray theory the energy is regarded as being transported along rays. The approximation of wave propagation, where one actually neglects the wave character, is often referred to as geometric optics. In the classic works of Brekhovskikh, 1960, Wait, 1962, and Baños, 1966, the geometric optics solutions are obtained when exact integral representations of the fields are evaluated by the asymptotic method of steepest descent. When the paths of integration are deformed into the paths of steepest descent, branch cuts and poles of the reflection and transmission coefficients may yield head waves (lateral waves) and channel waves (ducted wave-guide modes) in addition to the geometric optics contributions. Baños (1966) gives a comprehensive treatment of that approach applied to dipole radiation in the presence of an ocean/air interface and develops accurate expressions for all field components with a dipole source in different orientations and at different positions in the two regions.

In the following we demonstrate that we can treat asymptotic ray theory in a unified framework for both dielectric and conductive media and that the description simplifies in the two special cases of either high frequency and low conductivity or low frequency and high conductivity. We assume slow spatial variation of the medium properties and consider source-free regions. Then equations 5a and 5b are both reduced to homogeneous Helmholtz equations. Taking $\boldsymbol{\Psi}$ to represent either the electric field or the magnetic field, we get

$$\nabla^2 \boldsymbol{\Psi} + k^2(\mathbf{r}) \boldsymbol{\Psi} = \mathbf{0}. \tag{12}$$

We use the well-known solution ansatz

$$\boldsymbol{\Psi}(\mathbf{r}) = \exp[ik_0 W(\mathbf{r})] \sum_{m=0}^{N} \frac{\boldsymbol{\Psi}_m(\mathbf{r})}{(ik_0)^m}, \tag{13}$$

where all of the terms might be frequency dependent but where the spatial variation is described by the phase term $W(\mathbf{r})$ and the slowly varying amplitudes $\boldsymbol{\Psi}_m(\mathbf{r})$. In the sum we indicate an upper limit N because infinite asymptotic series usually diverge. The parameter k_0 is the wavenumber for a reference medium.

In our solution ansatz the underlying assumption is that only one geometric wavefront passes through each point in space (Born and Wolf, 1999). In regions where several rays pass through the same point in space, we often need to use a more general solution ansatz that contains sums over raypaths. A thorough discussion of an analogous case from seismic modeling can be found in Chapman (2004). Inserting our ansatz into equation 12 and solving for powers of ik_0 when the magnitude of k_0 becomes large, we get

$$(\nabla W)^2 = \left(\frac{k}{k_0}\right)^2, \tag{14a}$$

$$\nabla^2 W \boldsymbol{\Psi}_0 + 2(\nabla W \cdot \nabla) \boldsymbol{\Psi}_0 = \mathbf{0}, \tag{14b}$$

$$\nabla^2 W \boldsymbol{\Psi}_m + 2(\nabla W \cdot \nabla) \boldsymbol{\Psi}_m + \nabla^2 \boldsymbol{\Psi}_{m-1} = \mathbf{0} \quad m \geq 1, \tag{14c}$$

where it is implicit that k_0 and k must have the same order of magnitude. Equation 14a is the eikonal equation, and equation 14b is the transport equation. In the eikonal equation, k/k_0 is a normalized slowness. In optics this is equivalent to a refraction index $n = \sqrt{\mu_r \tilde{\varepsilon}_r} = k/k_0$, where μ_r and $\tilde{\varepsilon}_r$ denote relative permeability and permittivity, respectively, and it is common to use vacuum as the reference medium. If we use only the first term in the ray expansion, the eikonal equation describes the raypaths whereas the transport equation describes how the slow, geometric amplitude variations must be to satisfy energy conservation.

The eikonal equation 14a is complex and frequency dependent. However, in two cases of particular interest, it becomes real and frequency independent. In the asymptotic limit $x > 10$ (Figure 1) or in nonconducting media, the wavenumber reduces to $k(\mathbf{r}) = \omega\sqrt{\mu(\mathbf{r})\varepsilon(\mathbf{r})}$. With $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, where the index zero refers to values in a chosen reference medium, we see that $(\nabla W)^2 = \mu(\mathbf{r})\varepsilon(\mathbf{r})/(\mu_0\varepsilon_0)$. In the solution ansatz in equation 13, we can in this case alternatively choose $i\omega W'(\mathbf{r})$ in the exponential. Then the eikonal equation becomes $(\nabla W')^2 = \mu(\mathbf{r})\varepsilon(\mathbf{r})$, where $\nabla W'$ now describes slowness.

In conducting media, low-frequency signals are approximated well by the dispersion relation $k(\mathbf{r}) = \sqrt{i\omega\mu(\mathbf{r})\sigma(\mathbf{r})}$. This corresponds to the asymptotic limit $x < 0.1$ in Figure 1. With $k_0 = \sqrt{i\omega\mu_0\sigma_0}$ the eikonal equation 14a is real and frequency independent for this case as well: $(\nabla W)^2 = \mu(\mathbf{r})\sigma(\mathbf{r})/(\mu_0\sigma_0)$. In the solution ansatz in equation 13, we can in this case alternatively choose $-\sqrt{-i\omega}W''(\mathbf{r})$ in the exponential. Then the eikonal equation becomes $(\nabla W'')^2 = \mu(\mathbf{r})\sigma(\mathbf{r})$, which is the solution approach found by Virieux et al. (1994).

Layered media

In both geophysics and optics, it is often of interest to consider EM fields in layered media. Then the medium varies only in one direction, and the spatial components orthogonal to this direction are well suited for a Fourier expansion. We consider a stack of homogeneous layers and decompose the wavenumber into $k^2 = k_x^2 + k_y^2 + k_z^2$. We choose the variation to be along the z-direction. After the Fourier expansion, $\partial_x \to ik_x$ and $\partial_y \to ik_y$; each Fourier component represents a superposition of up- and downgoing plane waves in each layer. Introducing $p_x = k_x/\omega$ and $p_y = k_y/\omega$, which represent slownesses in the x- and y-directions, respectively, and following the formalism of Ursin (1983), we can express Maxwell's equations as

$$\partial_z \boldsymbol{\mathcal{B}} = -i\omega \boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{B}} + \boldsymbol{\mathcal{S}}, \tag{15}$$

with field vector $\boldsymbol{\mathcal{B}} = [E_x, E_y, -H_y, H_x]^T$, source vector $\boldsymbol{\mathcal{S}} = [p_x J_{0z}/\tilde{\varepsilon}, p_y J_{0z}/\tilde{\varepsilon}, J_{0x}, J_{0y}]^T$, and system matrix $\boldsymbol{\mathcal{A}} = [\mathbf{0}, \boldsymbol{\mathcal{A}}_1; \boldsymbol{\mathcal{A}}_2, \mathbf{0}]$, where

$$\boldsymbol{\mathcal{A}}_1 = \frac{1}{\tilde{\varepsilon}} \begin{pmatrix} \mu\tilde{\varepsilon} - p_x^2 & -p_x p_y \\ -p_x p_y & \mu\tilde{\varepsilon} - p_y^2 \end{pmatrix},$$

$$\boldsymbol{\mathcal{A}}_2 = \frac{1}{\mu} \begin{pmatrix} \mu\tilde{\varepsilon} - p_y^2 & p_x p_y \\ p_x p_y & \mu\tilde{\varepsilon} - p_x^2 \end{pmatrix}. \tag{16}$$

At interfaces we introduce mathematical idealizations that lead to discontinuities of the material parameters. The boundary conditions state continuity of $\boldsymbol{\mathcal{B}}$, which leads to the Fresnel reflection and transmission coefficients for two orthogonal states of polarization: transverse electric (TE) and transverse magnetic (TM). Explicit expressions for the Fresnel coefficients are derived in Appendix A. The propagator theory of multilayer systems can be used to compute the overall TE and TM reflection and/or transmission responses from several layers. One finally obtains the total response from the multilayered system by an inverse Fourier transform (Ward and Hohmann, 1987; Løseth, 2000). Most modeling codes for horizontally layered media are based on this theory; for low-frequency waves in conductive materials, the formulas in Chave and Cox (1982) are readily obtained from this formalism.

Green's functions

From Green's theorem stems the concept of Green's functions (Green, 1828), which define the impulse response of a medium. These functions can be used to solve inhomogeneous differential equations with boundary conditions. In EM theory they provide an alternative solution method to vector potential techniques. The dyadic Green's functions for the electric field $\mathbf{G}_E$ and magnetic field $\mathbf{G}_H = \nabla \times \mathbf{G}_E$ characterize the EM response resulting from a directional point source. Once the Green's functions are constructed, the EM field from a source distribution can be determined, and the electric and magnetic fields outside the source region are given as volume integrals over the source and their respective Green's function:

$$\mathbf{E}(\mathbf{r},\omega) = i\omega\mu \int_{V_0} d\mathbf{r}_0 \mathbf{G}_E(\mathbf{r},\omega,\mathbf{r}_0)\mathbf{J}_0(\mathbf{r}_0), \tag{17a}$$

$$\mathbf{H}(\mathbf{r},\omega) = \int_{V_0} d\mathbf{r}_0 \mathbf{G}_H(\mathbf{r},\omega,\mathbf{r}_0)\mathbf{J}_0(\mathbf{r}_0). \tag{17b}$$

The Green's functions for the EM field obey the reciprocity relation

$$\mathbf{G}(\mathbf{r},\omega,\mathbf{r}_0) = \mathbf{G}^T(\mathbf{r}_0,\omega,\mathbf{r}), \tag{18}$$

where $\mathbf{G}$ represents either $\mathbf{G}_E$ or $\mathbf{G}_H$ and where T denotes transpose. This means that the mth component of a signal at $\mathbf{r}$ caused by a unit impulse applied in the nth direction at $\mathbf{r}_0$ equals the nth component of a signal at $\mathbf{r}_0$ caused by a unit impulse applied in the mth direction at $\mathbf{r}$. The reciprocity relation (equation 18) then gives us the conditions for interchanging source and receiver without affecting the measured signal.

Analytic solution in a homogeneous medium

In homogeneous media, the terms containing derivatives of the medium parameters in equations 5a and 5b vanish. The electric and magnetic fields are then solutions of inhomogeneous Helmholtz equations where the wavenumber k, found in equation 3, is now space invariant. Following Tai (1994), the dyadic Green's functions for the electric and magnetic field are now solutions of

$$\nabla^2\mathbf{G}_E + k^2\mathbf{G}_E = -\left[\mathbf{I} + \frac{1}{k^2}\nabla\nabla\right]\delta(\mathbf{r}-\mathbf{r}_0), \tag{19a}$$

$$\nabla^2\mathbf{G}_H + k^2\mathbf{G}_H = -\nabla\times\left[\mathbf{I}\delta(\mathbf{r}-\mathbf{r}_0)\right]. \tag{19b}$$

Here, $\nabla\times$ denotes curl of a dyadic function, $\nabla\nabla$ is a dyadic operator, and $\mathbf{I}$ is the unit diagonal dyad. The Green's function solutions are

$$\mathbf{G}_E(\mathbf{r},\omega,\mathbf{r}_0) = \frac{\exp(ikr)}{4\pi r^3}\begin{bmatrix} h_1 + (x-x_0)^2h_2 & (x-x_0)(y-y_0)h_2 & (x-x_0)(z-z_0)h_2 \\ (x-x_0)(y-y_0)h_2 & h_1 + (y-y_0)^2h_2 & (y-y_0)(z-z_0)h_2 \\ (x-x_0)(z-z_0)h_2 & (y-y_0)(z-z_0)h_2 & h_1 + (z-z_0)^2h_2 \end{bmatrix}, \tag{20a}$$

$$\mathbf{G}_H(\mathbf{r},\omega,\mathbf{r}_0) = \frac{(ikr-1)\exp(ikr)}{4\pi r^3}\begin{bmatrix} 0 & -(z-z_0) & (y-y_0) \\ (z-z_0) & 0 & -(x-x_0) \\ -(y-y_0) & (x-x_0) & 0 \end{bmatrix}, \tag{20b}$$

where

$$r = |\mathbf{r}-\mathbf{r}_0| = \left[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right]^{1/2},$$

$$h_1 = r^2\left(1 - \frac{1}{ikr} - \frac{1}{(kr)^2}\right),$$

$$h_2 = \left(-1 + \frac{3}{ikr} + \frac{3}{(kr)^2}\right).$$

These equations are equivalent to the dyadic Green's functions found in Ward and Hohmann (1987), but here they are not restricted to a conductive medium. In the special case of a homogeneous medium, the Green's functions, in addition to obeying the general law of reciprocity (equation 18), obey the symmetry relation $\mathbf{G}_E(\mathbf{r},\omega,\mathbf{r}_0) = \mathbf{G}_E(\mathbf{r}_0,\omega,\mathbf{r})$ and $\mathbf{G}_H(\mathbf{r},\omega,\mathbf{r}_0) = -\mathbf{G}_H(\mathbf{r}_0,\omega,\mathbf{r})$.

DIPOLE RADIATION

We consider dipole radiation in a homogeneous medium to illustrate that the unified description in the frequency domain yields the correct expressions in the time domain for nondispersive waves in nonconductive media and low-frequency waves in conductive media.

An infinitesimal electric dipole antenna can be represented by a periodic line current of length $l \ll \lambda$ with current amplitude $I(\omega)$. This gives the dipole current moment Il. For simplicity and clarity we use spherical coordinates with the source located at the origin and pointing in the x-direction. Then the source-current density becomes $\mathbf{J}_0 = Il\delta(\mathbf{r})\hat{\mathbf{x}}$. Let θ denote the angle between the x-axis and the radius vector $\mathbf{r}$, and let ϕ denote the angle between the y-axis and a projection of $\mathbf{r}$ into the plane defined by the y- and z-axes. The source and coordinate systems are shown in Figure 2. The EM field in the frequency domain from such a source-current distribution is easily found with the aid of the Green's functions in equations 20a and 20b and the relation between the fields and their Green's functions (equa-

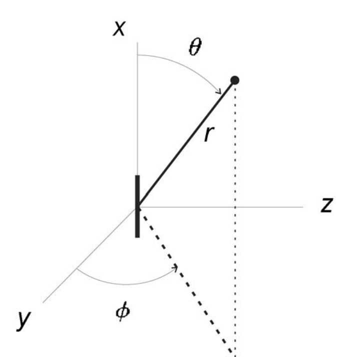

Figure 2. The Cartesian and spherical coordinate systems are shown with the dipole antenna included. The antenna is oriented in the x-direction. The elevation angle θ is between the plane defined by the y- and z-axes and the x-axis (yz-plane). The azimuth angle ϕ is in the yz-plane.

tions 17a and 17b). After a transform from Cartesian to spherical coordinates, the radiated dipole field is expressed as

$$\mathbf{E}(\mathbf{r},\omega) = \frac{ik\eta Il}{4\pi r}\exp(ikr)\left[-\left(1-\frac{1}{ikr}+\frac{1}{(ikr)^2}\right)\hat{\boldsymbol{\theta}}\sin\theta + \left(\frac{1}{ikr}-\frac{1}{(ikr)^2}\right)2\hat{\mathbf{r}}\cos\theta\right], \tag{21a}$$

$$\mathbf{H}(\mathbf{r},\omega) = \frac{ikIl}{4\pi r}\exp(ikr)\left(1-\frac{1}{ikr}\right)(-\hat{\boldsymbol{\phi}}\sin\theta), \tag{21b}$$

where $\omega\mu = k\eta$ and where η is the impedance as defined in equation 4. The exact expressions for a Hertzian dipole are excellent approximations for the fields from a physical dipole at distances $r \gg l$ and are found in a variety of works (see Burrows, 1978, and Baños, 1966). The magnetic field is circulating around the axis (in the $\hat{\boldsymbol{\phi}}$ direction), the electric field is in the plane defined by the axis and the radial distance (in the $\mathbf{r}$ and $\hat{\boldsymbol{\theta}}$ directions), and the signal level is determined by the dipole current moment Il. The radiation pattern is rotationally symmetric about the dipole axis, and the maximum radiation is in the normal direction ($\theta = 90°$). A more thorough investigation of the radiation pattern can be found in Appendix B. We observe that the exact dipole formulas in equations 21a and 21b are ray-series expansions of the same kind as in equation 13. In this case the ray-series method actually yields exact results with $N = 2$ for the electric field and $N = 1$ for the magnetic field.

Time-domain signals

The EM field in the time domain from an electric dipole is found by applying an inverse Fourier transform of the frequency-domain fields in equations 21a and 21b:

$$\mathbf{e}(\mathbf{r},t) = \frac{\mu}{4\pi r}\left\{e_0(r,t)(\hat{\boldsymbol{\theta}}\sin\theta) + \left[e_1(r,t) + e_2(r,t)\right](2\hat{\mathbf{r}}\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta)\right\}, \tag{22a}$$

$$\mathbf{h}(\mathbf{r},t) = \frac{1}{4\pi r}\left\{h_0(r,t) + h_1(r,t)\right\}(\hat{\boldsymbol{\phi}}\sin\theta), \tag{22b}$$

where the real electric field is represented by $\mathbf{e}(\mathbf{r},t)$ and the real magnetic field by $\mathbf{h}(\mathbf{r},t)$. The two far-field terms are

$$e_0(r,t) = -\frac{l}{2\pi}\int_{-\infty}^{\infty}\omega I(\omega)\mathrm{e}^{i(kr-\omega t)}\mathrm{d}\omega,$$

$$h_0(r,t) = -\frac{l}{2\pi}\int_{-\infty}^{\infty}ikI(\omega)\mathrm{e}^{i(kr-\omega t)}\mathrm{d}\omega. \tag{23}$$

The electric near-field terms $e_1(r,t)$ and $e_2(r,t)$ contain the two higher-order terms in the dipole expansion. The two integrals in this expansion are given by consecutive multiplication by the factor $-1/(ikr)$ of the far-field term. The magnetic near-field term $h_1(r,t)$ contains one integral that equals the far-field term times this factor. Note that the additional factor $1/(ik)$ in the integrals in equation 23 equals an integration that can be carried out in the time domain:

$$\int_{-\infty}^{\infty}\frac{f(\omega)}{ik}e^{ikr}\mathrm{d}\omega = -\int_{r}^{\infty}\int_{-\infty}^{\infty}f(\omega)e^{ikr'}\mathrm{d}\omega\mathrm{d}r'. \tag{24}$$

Let us consider the radiated EM field from an electric dipole in a nonconductive medium and a conductive medium. We first look at the resulting EM field from a simple time harmonic current source $I(t) = I_0\cos(\omega_0 t)$ and next derive the step response, i.e., the EM field from a constant current turned on at $t = 0$. The frequency-domain representations of the two current distributions are $I(\omega) = I_0\pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$ and $I(\omega) = I_0[\pi\delta(\omega) - (1/i\omega)]$, where $\delta(\omega)$ represents the Dirac delta function.

Nondispersive and nonconductive media

For nondispersive waves in nonconducting materials, we have the dispersion relation $k(\omega) = \omega/c$, where $c = 1/\sqrt{\mu\varepsilon}$ is the velocity in the medium. The radiated EM field from a dipole source for a time harmonic current distribution then becomes

$$\mathbf{e}(\mathbf{r},t) = \frac{\mu\omega_0 I_0 l\sin\varphi}{4\pi r}\hat{\boldsymbol{\theta}}\sin\theta + \frac{I_0 l}{4\pi\varepsilon r^2}\left[\frac{1}{c}\cos\varphi - \frac{1}{\omega_0 r}\sin\varphi\right](\hat{\boldsymbol{\theta}}\sin\theta + 2\hat{\mathbf{r}}\cos\theta), \tag{25a}$$

$$\mathbf{h}(\mathbf{r},t) = \frac{I_0 l}{4\pi r}\left[\frac{\omega_0}{c}\sin\varphi + \frac{1}{r}\cos\varphi\right]\hat{\boldsymbol{\phi}}\sin\theta, \tag{25b}$$

where $\varphi = \omega_0\left(\frac{r}{c} - t\right)$.

In the step-response calculations we readily obtain the well-known result with a perfect but delayed delta pulse:

$$\mathbf{e}(\mathbf{r},t) = \frac{\mu I_0 l}{4\pi r}\delta\left(t-\frac{r}{c}\right)\hat{\boldsymbol{\theta}}\sin\theta + \frac{I_0 lt}{4\pi\varepsilon r^3}H\left(t-\frac{r}{c}\right)(\hat{\boldsymbol{\theta}}\sin\theta + 2\hat{\mathbf{r}}\cos\theta), \tag{26a}$$

$$\mathbf{h}(\mathbf{r},t) = \frac{I_0 l}{4\pi r}\left[\frac{1}{c}\delta\left(t-\frac{r}{c}\right) - \frac{1}{r}H\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{\phi}}\sin\theta, \tag{26b}$$

where $H(t)$ is the Heaviside step function. Note that the electric field has a near-field contribution only for $t \geq r/c$. This is a contribution equal to the static dipole field from the charges $\pm q_0 = \pm I_0 t$ accumulated on the dipole ends in a nonconductive material. For the magnetic field, the first term is the radiated far-field pulse, but for $t \geq r/c$ the near-field term yields a static magnetic field caused by the constant source current. The far-field term has the same time dependence as the electric field since the impedance η is independent of frequency in nonconductive materials.

Conductive media

For low frequencies and conductive materials we use the dispersion relation $k(\omega) = (1+i)\sqrt{\omega\mu\sigma/2}$. A time harmonic source current then yields

$$\mathbf{e}(\mathbf{r},t) = \frac{\mu\omega_0 I_0 l}{4\pi r}\mathrm{e}^{-\beta r}\sin(\varphi_c)(\hat{\boldsymbol{\theta}}\sin\theta) + \frac{I_0 l}{4\pi r^2}\mathrm{e}^{-\beta r}\left[\sqrt{\frac{\mu\omega_0}{\sigma}}\cos\left(\varphi_c - \frac{\pi}{4}\right) + \frac{1}{\sigma r}\cos\varphi_c\right](2\hat{\mathbf{r}}\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta), \quad (27a)$$

$$\mathbf{h}(\mathbf{r},t) = \frac{I_0 l}{4\pi r}\mathrm{e}^{-\beta r}\left[\sqrt{\mu\sigma\omega_0}\cos\left(\varphi_c - \frac{\pi}{4}\right) + \frac{1}{r}\cos\varphi_c\right](\hat{\boldsymbol{\phi}}\sin\theta), \quad (27b)$$

where $\varphi_c = (\beta r - \omega_0 t)$ and $\beta = \sqrt{\omega_0\mu\sigma/2}$.

When calculating the step response, we evaluate the integral asymptotically. That is, since the main contribution to the integral comes from the low-frequency regime, we approximate the integral by integrating up to a cutoff frequency where the low-frequency wavenumber approximation for conductive media is valid. We then use this approximation and expand the integration limits to infinity again. The approximation is justified by the heavy attenuation of the higher frequencies in highly conductive media, which implies that the measurable part of the signal is in the low-frequency region. A formal correct mathematical treatment should include the entire dispersion relation given in equation 3. This would lead to a solution containing weighted sums of heavily attenuated delta pulses for higher frequencies. A discussion of pulse propagation in dispersive media can be found in the classic papers of Sommerfeld (1914) and Brillouin (1914). A thorough treatment is also given in Stratton (1941). Morse and Feshbach (1953) solve the expression in equation 23 and the higher-order terms for the complete dispersion relation. However, our goal here is to demonstrate the calculation that leads to the quasi-static approximation.

To solve the first integral in equation 23 for the step response, we introduce a new variable $x = \sqrt{\omega t/2}$ and use $\cos(-x) = \cos(x)$. Then

$$e_0(r,t) \cong \frac{4I_0 l}{\pi t}\int_0^{\infty} x\exp\left(-\sqrt{\frac{\mu\sigma r^2}{t}}x\right)\cos\left[\sqrt{\frac{\mu\sigma r^2}{t}}x - 2x^2\right]\mathrm{d}x. \quad (28)$$

This integral is tabulated in equation 3.966.2 in Gradshteyn and Ryzhik (1980). The result is

$$e_0(r,t) \cong I_0 l\sqrt{\frac{\mu\sigma r^2}{4\pi t^3}}\exp\left(-\frac{\mu\sigma r^2}{4t}\right). \quad (29)$$

For the higher-order terms that constitute the near field, we use the method in equation 24. We then obtain by repeated integration

$$e_1(r,t) \cong \frac{I_0 l}{r\sqrt{\pi\mu\sigma t}}\exp\left(-\frac{\mu\sigma r^2}{4t}\right),$$

$$e_2(r,t) \cong \frac{I_0 l}{\mu\sigma r^2}\left[1 - \mathrm{erf}\left(\sqrt{\frac{\mu\sigma r^2}{4t}}\right)\right], \quad (30)$$

where the error function is

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-x^2)\mathrm{d}x. \quad (31)$$

To determine the magnetic field, we note that since we are using the dispersion relation $k^2 = i\omega\mu\sigma$, the terms in the expression for the magnetic field only differ from the terms in the one higher-order term of the electric field by the factor σr.

For convenience we express the dipole step response in terms of a scaled time $x = t/t_d$, where $t_d = \mu\sigma r^2/4$. In terms of these parameters, the EM field becomes

$$\mathbf{e}(\mathbf{r},t) \cong \frac{I_0 l}{4\pi\sigma r^3}\left[f_1\left(\frac{t}{t_d}\right)\hat{\boldsymbol{\theta}}\sin\theta + f_2\left(\frac{t}{t_d}\right)(\hat{\boldsymbol{\theta}}\sin\theta + 2\hat{\mathbf{r}}\cos\theta)\right], \quad (32a)$$

$$\mathbf{h}(\mathbf{r},t) \cong \frac{I_0 l}{4\pi r^2}f_2\left(\frac{t}{t_d}\right)\hat{\boldsymbol{\phi}}\sin\theta, \quad (32b)$$

where

$$f_1(x) = \frac{4}{\sqrt{\pi x^3}}\exp\left(-\frac{1}{x}\right) \quad (33)$$

is the far-field step response for the electric field and

$$f_2(x) = 1 + \frac{2}{\sqrt{\pi x}}\exp\left(-\frac{1}{x}\right) - \mathrm{erf}\left(\frac{1}{\sqrt{x}}\right) \quad (34)$$

is the step response of the two near-field terms for the electric field. These step responses are shown in Figure 3. The step response for the magnetic field is given by $f_2(x)$. If we split $f_2(x)$ into the far-field and near-field terms for the magnetic field, we get a relation between

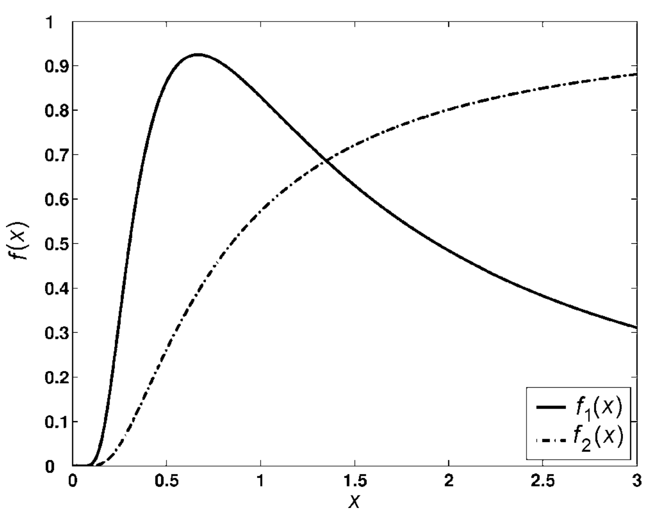

Figure 3. Step responses for dipole radiation in conductive materials: far-field response $f_1(x)$ (solid line) and near-field response $f_2(x)$ (broken line). The far-field response is largest for $x < 1.4$.

the two components that is similar to the relation between $f_1(x)$ and $f_2(x)$. To illustrate our point, it is sufficient to consider $f_1(x)$ and $f_2(x)$. When doing so, we see that although the far-field term arrives first and dominates for $t < t_d$, we cannot neglect the near-field term. We also have the same geometric r dependence for both the near field and the far field. In contrast to the frequency domain, there is no clear distinction between the far field and the near field. In a conductive material, for $t \gg t_d$, the near-field term yields a constant, static dipole field:

$$\mathbf{e}_{\mathrm{dc}}(\mathbf{r}) = \frac{I_0 l}{4\pi\sigma r^3}(\hat{\boldsymbol{\theta}}\sin\theta + 2\hat{\mathbf{r}}\cos\theta),$$

$$\mathbf{h}_{\mathrm{dc}}(\mathbf{r}) = \frac{I_0 l}{4\pi r^2}\hat{\boldsymbol{\phi}}\sin\theta. \tag{35}$$

With a current impulse $q_0\delta(t)$ of total charge q_0 at $t = 0$, the frequency-domain current amplitude is $I_0 = q_0$. The resulting impulse response is obtained from the step response by a simple time differentiation:

$$\mathbf{e}(\mathbf{r},t) \cong \frac{q_0 l}{\pi\mu\sigma^2 r^5}\left[g_1\left(\frac{t}{t_d}\right)\hat{\boldsymbol{\theta}}\sin\theta + g_2\left(\frac{t}{t_d}\right)(\hat{\boldsymbol{\theta}}\sin\theta + 2\hat{\mathbf{r}}\cos\theta)\right], \tag{36a}$$

$$\mathbf{h}(\mathbf{r},t) \cong \frac{q_0 l}{\pi\mu\sigma r^4} g_2\left(\frac{t}{t_d}\right)\hat{\boldsymbol{\phi}}\sin\theta, \tag{36b}$$

where $g_i = df_i(x)/dx$ $(i = 1,2)$ are the derivatives of the functions in equations 33 and 34. These time responses are shown in Figure 4. We see that the far-field response arrives first and has a peak value more than three times that of the near-field response, but at later times the near-field term cannot be neglected.

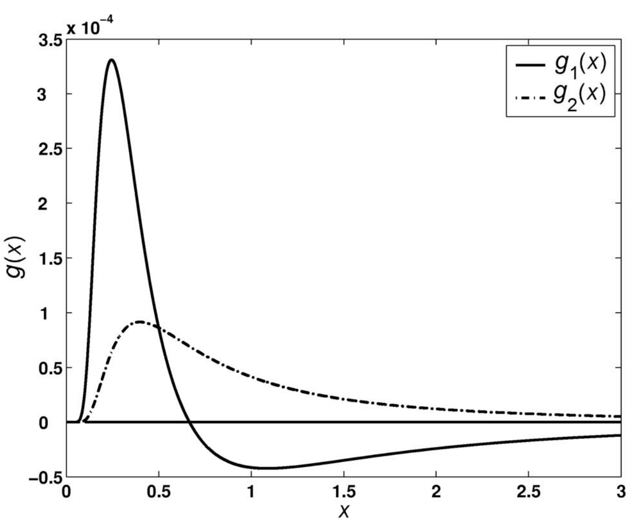

Figure 4. Impulse responses for dipole radiation in conductive materials: far-field response $g_1(x)$ (solid line) and near-field response $g_2(x)$ (broken line). The far-field response is largest for $x < 0.5$.

DISCUSSION

In SBL and similar applications of marine controlled-source EM fields, the signal sources are towed electrical dipole antennas, and they are very well approximated by Hertzian dipoles at the frequencies and wavelengths involved. Detectable signal transmission is obtained only at very low frequencies, $\omega \ll \omega_0 = \sigma/\varepsilon$. In this limit the contribution from the displacement current can be ignored, and equations 27a and 27b describe propagation of single-frequency components in homogeneous media. Compared to time-harmonic signal propagation in nonconductive media (equations 25a and 25b), the propagation of a low-frequency signal in conductive materials is characterized by the damping term and the frequency-dependent phase velocity. In addition the phase behavior between the electric and magnetic fields differs in conductive media. The wavelength is $\lambda = 2\pi\delta$; thus, we have 54.6 dB attenuation per wavelength, and in most cases it is only possible to detect signals that are transmitted a few wavelengths. One then normally wants to use very low frequencies and long wavelengths ($\lambda > 1$ km) to reach down to deeply buried layers.

In exploration configurations where one uses transient source signals, one gets responses that resemble the strongly distorted pulse forms in Figures 3 and 4. The propagation has the same characteristics as in many diffusion processes. The far-field and near-field terms in equations 32a and 36a have the same geometric r-dependence. The difference between the time-domain step responses in nonconductive and conductive materials is caused by the strong dispersion and frequency-dependent attenuation in conductive materials.

We observe that the frequency-domain treatment of signal propagation in homogeneous media leads to the correct mathematical description of time harmonic signals and transients for both conductive and nonconductive media. A correct time-domain approach would of course lead to the same equations. If we derive the wave equation from Maxwell's equations in the time domain, assuming permeability, permittivity, and conductivity are independent of frequency, we get the following damped wave equation both for the electric and the magnetic fields when we ignore the source term:

$$\nabla^2\boldsymbol{\psi} = \mu\sigma\frac{\partial\boldsymbol{\psi}}{\partial t} + \mu\varepsilon\frac{\partial^2\boldsymbol{\psi}}{\partial t^2}. \tag{37}$$

The term involving conductivity represents a damping term in the wave equation. Without damping, we would have the wave equation

$$\nabla^2\boldsymbol{\psi} = \mu\varepsilon\frac{\partial^2\boldsymbol{\psi}}{\partial t^2}, \tag{38}$$

which describes nondispersive waves in nonconductive materials. If the damping term becomes completely dominant, as is the case for low-frequency signals in conductive materials, the wave equation is well approximated by

$$\nabla^2\boldsymbol{\psi} = \mu\sigma\frac{\partial\boldsymbol{\psi}}{\partial t}, \tag{39}$$

which is the diffusion equation one gets if the displacement current in Maxwell's equations is ignored, i.e., the speed of light is assumed infinite. This is often referred to as the quasi-static limit (e.g., Jackson, 1998). The equation has the same form as diffusion equations found in various literature (e.g., Crank, 1975).

The hyperbolic wave equation and the parabolic diffusion equation are both transformed into an elliptic equation when moving from the time domain into the frequency domain (Sommerfeld, 1967). In the frequency domain, the propagation is characterized by the position of k in the complex plane. The wavenumber might vary from the real axis to a line rotated by 45° with the real axis. The first case corresponds to propagation of an undamped wave, whereas the second case represents highly attenuated propagating waves. Between these two extreme cases, there is a gradual change from undamped wave propagation to highly attenuated wave propagation or diffusion. Thus, the diffusion equation has wave solutions. Moreover, diffusion equation 39 is a vector equation. Depending on what one means by a diffusion process, one should be careful about thinking of the physical process as a diffusion process since the notion of diffusion often is characterized by random motion, which constitutes a probability distribution that describes diffusive transport (see Einstein, 1905, on Brownian motion). The conservation of direction and polarization of the EM field might not easily be related to this physical picture (as in Milne's problem, Morse and Feshbach, 1953).

On the other hand, when one thinks of the propagation of low-frequency fields in conductive media in terms of waves, one must consider that these waves are strongly attenuated and highly dispersive. Thus, the concept of time reversal, which is often used in processing of seismic data (Claerbout, 1971), cannot be applied directly. Moreover, the concept of group velocity loses its traditional significance in this case (Stratton, 1941).

As stated above, there is nothing wrong in using equation 39 as the starting point for treating low-frequency EM fields in conducting media (assuming frequency-independent material parameters). This quasi-static approach is often used in connection with low-frequency EM fields in conductive media. The concept of looking at field propagation in terms of currents follows from this. However, we have also seen that by considering EM fields in the frequency domain, we can treat wave propagation in both nonconducting media and conducting media. Thus, the two apparently very different cases of nondispersive wave propagation and low-frequency, highly dispersive wave propagation can be treated within a unified mathematical framework. In fact, all of the well-established tools of wave theory can be applied directly. Moreover, there is no clear transition zone from one process type to the other, as can be observed from Figure 1. An example of a unified treatment is found in modeling EM wave propagation in layered media. The well-known description of reflection and refraction of plane EM waves at planar interfaces and the associated division of the fields into TE and TM modes implies that both nondispersive waves in nonconductive materials and low-frequency waves in conductive materials obey the same equations (see Appendix A). Thus, layers that are dominantly dispersive and layers that are dominantly dissipative can be treated on an equal footing. Within this picture the EM response from buried highly resistive layers can be explained in terms of the characteristic difference between TE and TM polarization. In SBL, this characteristic difference is used to detect buried hydrocarbon layers by orienting the dipole source and receiver antennas in specific directions. However, in many other applications that use low-frequency EM fields, it might be advantageous to consider the problem from the quasi-static point of view.

SUMMARY

The basic theory of EM wave propagation has been reviewed and used to develop a unified frequency-domain description that applies for nondispersive waves in nonconductive materials and for highly dispersive, low-frequency waves in conductive materials.

We have considered the time-domain responses for an infinitesimal electric dipole antenna and have shown that a unified description in the frequency domain yields both the undistorted pulses in nonconductive materials and the highly distorted diffusive pulses for low-frequency signals in conductive materials. In the latter case both the step response and the impulse response are strongly attenuated and distorted.

The question of whether EM field propagation in conductive materials can be referred to as diffusion or wave propagation has been discussed. We have shown that the approximation that results in a diffusion-like equation is valid. We have also shown that the wave-propagation description provides the correct mathematical formulation. We conclude that one might call low-frequency propagation of EM fields in conductive media what one prefers. But when one characterizes field propagation as diffusion, it might be clearer to add that one is not referring to the random motion usually affiliated with diffusion processes. When field propagation is characterized as wave propagation, one should remember that waves are highly dispersive and strongly attenuated.

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APPENDIX A

REFLECTION AND REFRACTION OF PLANE WAVES AT PLANAR INTERFACES

At an interface between two homogeneous media, the boundary conditions for the tangential components of **E** and **H** become (Stratton, 1941)

$$\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0, \quad \mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K},$$

where **K** is a surface current and **n** is a unit normal vector to the surface. The subscripts 1 and 2 denote the fields in medium 1 and medium 2, respectively. When the conductivities of the media are finite, there is no surface current, and we may assume that the tangential components of both **E** and **H** are continuous.

Now, consider plane waves impinging at a planar interface. As depicted in Figure A-1, we denote the incoming, reflected, and transmitted electric and magnetic fields with the subscripts 1, 1′, and 2, respectively. Phase-matching conditions at the interface give Snell's law and the law of reflection since the incoming, reflected, and transmitted fields must have equal phases at the interface. These laws can be expressed as

$$\sin\theta_1 = \sin\theta_{1'}, \tag{A-2a}$$

$$k_1 \sin\theta_1 = k_2 \sin\theta_2, \tag{A-2b}$$

where $\theta_1 = \theta_{1'}$ denotes the angle between the incoming (reflected) ray and interface normal and where θ_2 denotes the angle between the transmitted ray and the opposite direction of the interface normal.

The wavenumbers in medium 1 and medium 2 are denoted by k_1 and k_2, respectively. When considering the relations between the amplitudes of the incident, reflected, and transmitted fields, we get

$$\mathbf{n} \times (\mathbf{E}_1 + \mathbf{E}_{1'}) = \mathbf{n} \times \mathbf{E}_2, \tag{A-3a}$$

$$\mathbf{n} \times (\mathbf{H}_1 + \mathbf{H}_{1'}) = \mathbf{n} \times \mathbf{H}_2. \tag{A-3b}$$

Now we resolve the electric field into one component that is normal to the plane of incidence. This component is parallel to the interface and is known in optics as s-polarization (Vašíček, 1960). Here we refer to it as the TE component (Born and Wolf, 1999). (Note that the terms transverse electric wave and transverse magnetic wave have different meanings in the theory of wave guides.) For isotropic media this leads to the relations

$$\mathbf{E}_{1'} = \frac{\mu_2 k_1 \cos\theta_1 - \mu_1 k_2 \cos\theta_2}{\mu_2 k_1 \cos\theta_1 + \mu_1 k_2 \cos\theta_2}\mathbf{E}_1, \tag{A-4a}$$

$$\mathbf{E}_2 = \frac{2\mu_2 k_1 \cos\theta_1}{\mu_2 k_1 \cos\theta_1 + \mu_1 k_2 \cos\theta_2}\mathbf{E}_1. \tag{A-4b}$$

For the other component the electric field is in the plane of incidence. Thus, the magnetic field is normal to the plane of incidence and is parallel to the interface. This is referred to as p polarization or the TM component. For isotropic media this leads to relations in terms of the magnetic field:

$$\mathbf{H}_{1'} = \frac{\tilde{\varepsilon}_2 k_1 \cos\theta_1 - \tilde{\varepsilon}_1 k_2 \cos\theta_2}{\tilde{\varepsilon}_2 k_1 \cos\theta_1 + \tilde{\varepsilon}_1 k_2 \cos\theta_2}\mathbf{H}_1, \tag{A-5a}$$

$$\mathbf{H}_2 = \frac{2\tilde{\varepsilon}_2 k_1 \cos\theta_1}{\tilde{\varepsilon}_2 k_1 \cos\theta_1 + \tilde{\varepsilon}_1 k_2 \cos\theta_2}\mathbf{H}_1. \tag{A-5b}$$

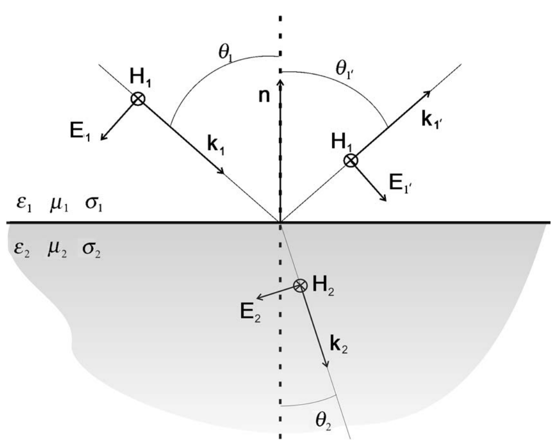

Figure A-1. Reflection and transmission of a plane wave described by rays at a planar interface. The wavenumber vector $\mathbf{k}$, electric field $\mathbf{E}$, and magnetic field $\mathbf{H}$ form a right-handed system. For the TE mode the electric field is perpendicular to the plane of incidence; for the TM mode, illustrated here, the magnetic field is perpendicular to the plane of incidence. The incoming electric and magnetic fields have subscript 1, the reflected electric and magnetic fields have subscript $1'$, and the transmitted electric and magnetic fields have subscript 2. The angle of incidence is denoted θ_1, the angle of reflection equals the angle of incidence and is denoted $\theta_{1'}$, and the refracted angle is denoted θ_2. The wavenumber is $k_1 = k_{1'}$ in medium 1 and k_2 in medium 2. The unit vector $\mathbf{n}$ is normal to the interface.

As seen from Figure A-1, we have $k_{z1} = k_1 \cos\theta_1$ and $k_{z2} = k_2 \cos\theta_2$. In general, $k_z = \sqrt{k^2 - k_x^2 - k_y^2}$, where the condition placed upon the double-valued root is $\mathrm{Im}(k_z) > 0$. We furthermore observe that the reflected tangential components of the electric and magnetic fields have opposite signs. We then derive the following reflection and transmission coefficients for TE and TM polarization:

$$r_{\mathrm{TE}} = \frac{\mu_2 k_{z1} - \mu_1 k_{z2}}{\mu_2 k_{z1} + \mu_1 k_{z2}}, \quad r_{\mathrm{TM}} = \frac{\tilde{\varepsilon}_1 k_{z2} - \tilde{\varepsilon}_2 k_{z1}}{\tilde{\varepsilon}_1 k_{z2} + \tilde{\varepsilon}_2 k_{z1}}, \tag{A-6a}$$

$$t_{\mathrm{TE}} = \frac{2\mu_2 k_{z1}}{\mu_2 k_{z1} + \mu_1 k_{z2}}, \quad t_{\mathrm{TM}} = \frac{2\tilde{\varepsilon}_2 k_{z1}}{\tilde{\varepsilon}_1 k_{z2} + \tilde{\varepsilon}_2 k_{z1}}. \tag{A-6b}$$

These coefficients are valid for both conductive and nonconductive media. The absolute value of the squared reflection coefficient represents reflected energy, whereas the absolute value of the squared transmission coefficient represents transmitted energy. Snell's law describes ray propagation across interfaces, and the law of reflection describes ray propagation reflected at an interface. In the general case, Snell's law describes a relation between complex quantities. However, in the two cases of nondispersive waves in nonconductive media and low-frequency waves in conductive media, Snell's law is a relation between real quantities.

APPENDIX B

RADIATION PATTERN

The complex Poynting vector $\mathbf{S}$ is defined as (Stratton, 1941; Jackson, 1998)

$$\mathbf{S} = \frac{1}{2}\mathbf{E} \times \mathbf{H}^*. \tag{B-1}$$

The time-averaged power density in a harmonic EM field is

$$\bar{\mathbf{S}} = \frac{1}{2}\,\mathrm{Re}\big(\mathbf{E} \times \mathbf{H}^*\big). \tag{B-2}$$

The magnetic field from the electric dipole considered in this paper has a component in the $\hat{\boldsymbol{\phi}}$ direction only. To determine the radiation pattern of the dipole, we need to evaluate the complex Poynting vector

$$\mathbf{S} = \frac{1}{2}\Big\{E_\theta H_\phi^* \hat{\mathbf{r}} - E_r H_\phi^* \hat{\boldsymbol{\theta}}\Big\}. \tag{B-3}$$

Using equations 21a and 21b, the products in equation B-3 become

$$E_\theta H_\phi^* = \frac{kk^*\eta|Il|^2}{(4\pi r)^2}\mathrm{e}^{-2\,\mathrm{Im}(k)r}\left[1 + \frac{i(k^* - k)}{kk^* r} + \frac{k - k^*}{k^2 k^* r^2} + \frac{i}{k^2 k^* r^3}\right]\sin^2\theta, \tag{B-4a}$$

$$E_r H_\phi^* = -\frac{kk^*\eta|Il|^2}{(4\pi r)^2} e^{-2\,\mathrm{Im}(k)r}\frac{1}{ikr}\left[1 + \frac{i(k^* - k)}{kk^* r} + \frac{1}{kk^* r^2}\right]\sin 2\theta, \qquad \text{(B-4b)}$$

where $|Il|^2 = II^* l^2$ is the absolute value of the dipole current moment; k and k^* are the wavenumber and its complex conjugate, respectively; and η is the impedance. In a nonconductive medium where $k = \omega\sqrt{\mu\varepsilon}$, the complex Poynting vector becomes

$$\mathbf{S} = \frac{k^2\eta|Il|^2}{32\pi^2}\frac{\sin^2\theta}{r^2}\hat{\mathbf{r}} - i\frac{k\eta|Il|^2}{32\pi^2}\frac{\sin 2\theta}{r^3}\hat{\boldsymbol{\theta}} + i\frac{\eta|Il|^2}{32\pi^2 k}\frac{1}{r^5}\left(\sin^2\theta\hat{\mathbf{r}} - \sin 2\theta\hat{\boldsymbol{\theta}}\right). \qquad \text{(B-5)}$$

Using $\sqrt{\mu\varepsilon} = 1/c$, where c is the velocity in the medium, the time-averaged power density becomes

$$\bar{\mathbf{S}} = \frac{\omega^2\mu|Il|^2}{32\pi^2 c}\frac{\sin^2\theta}{r^2}\hat{\mathbf{r}}. \qquad \text{(B-6)}$$

When considering low-frequency radiation in conductive media, we use the dispersion relation $k = (1 + i)\sqrt{(\mu\omega\sigma)/2}$. The complex Poynting vector in this case is

$$\mathbf{S} = \frac{\omega\mu\beta|Il|^2}{32\pi^2}\frac{e^{-2\beta r}}{r^2}\left\{\left[1 + \frac{1}{\beta r} + \frac{1}{\beta^2 r^2}\right]\sin^2\theta\hat{\mathbf{r}} - \left[\frac{1}{\beta r} + \frac{1}{2\beta^3 r^3}\right]\sin 2\theta\hat{\boldsymbol{\theta}}\right\} - i\frac{\omega\mu\beta|Il|^2}{32\pi^2}\frac{e^{-2\beta r}}{r^2}\left\{\left[\frac{1}{\beta r} + \frac{1}{\beta^2 r^2} - \frac{1}{2\beta^3 r^3}\right]\sin^2\theta\hat{\mathbf{r}} + \frac{\sin 2\theta}{\beta^2 r^2}\hat{\boldsymbol{\theta}}\right\}, \qquad \text{(B-7)}$$

where $\beta = \sqrt{\omega\mu\sigma/2}$. The real part of the Poynting vector now becomes

$$\bar{\mathbf{S}} = \frac{\omega\mu\beta|Il|^2}{32\pi^2}\frac{e^{-2\beta r}}{r^2}\left\{\left[1 + \frac{1}{\beta r} + \frac{1}{\beta^2 r^2}\right]\sin^2\theta\hat{\mathbf{r}} - \left[\frac{1}{\beta r} + \frac{1}{2\beta^3 r^3}\right]\sin 2\theta\hat{\boldsymbol{\theta}}\right\}. \qquad \text{(B-8)}$$

Observe that the conductive case yields an attenuation term as well as terms for higher negative powers of r. Note also that the time-averaged power density has a component in the $\hat{\boldsymbol{\theta}}$ direction.

In the nonconductive case, the total radiated power P_r is calculated by integrating $\hat{\mathbf{r}} \cdot \bar{\mathbf{S}}$ over a sphere of radius r. The integration is straightforward and gives

$$P_r = \int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} d\theta r^2 \sin\theta \bar{S}_r = \frac{\omega^2\mu|Il|^2}{12\pi c}. \qquad \text{(B-9)}$$

If we normalize the power density $\bar{S}_r(\theta, \phi)$ on the total radiated power averaged over all angles, we get the directive gain $G(\theta, \phi)$, where

$$G(\theta, \phi) = \frac{\bar{S}_r}{\dfrac{P_r}{4\pi r^2}} = \frac{3}{2}\sin^2\theta. \qquad \text{(B-10)}$$

A plot of the directive gain is shown in Figure B-1. For the conductive case the presence of absorption represented by the attenuation term indicates that we cannot find the total radiated power by integrating over a sphere. In this case, one would rather consider the time-averaged power density at the coordinates of interest. However, the radiation pattern, normally understood to describe the pattern of the radiated power in the far field can still be represented by Figure B-1.

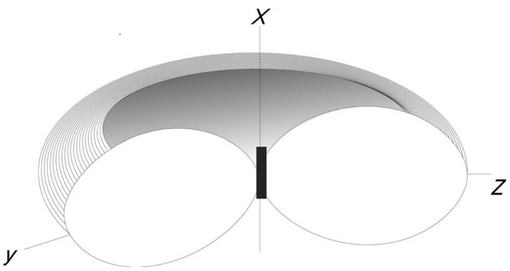

Figure B-1. Radiation pattern for the electric dipole. The radiation is symmetric around the dipole axis and largest normal to the dipole axis.

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